ANALOG AND DIGITAL SIMULATIONS OF MULTIPLEX SYSTEM PERFORMANCE

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ABSTRACT

Generalized multiplexing system models allow an additional degree of flexibility for the communication system designer. For channels with a single constraint, such as limited bandwidth, it is possible to determine optimum multiplexing systems. For more complicated channels with a combination of constraints, simulation programs are often the best way to select the best multiplexing system and to determine its performance. Results are presented for both analog and digital simulations as an example of the procedures involved. The system used as an example is based on the real exponential set of orthonormal waveforms.

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Introduction

Multiplexing is the process by which several message waveforms are combined into a single function of time for transmission over a single channel. The conventional methods, time and frequency division multiplexing, are but two of the many possible. In 1951 Marchand and Holloway proposed a systematic method for the development of other types of multiplexing systems by the use of general orthogonal functions. This method was based on the fact that the separability of the message channels in time and frequency multiplexing utilized the property of orthogonality of nonoverlapping time pulses and sine waves of different frequencies respectively. Since many other functions have the property of orthogonality, many other types of multiplexing systems are possible. A paper by Zadeh and Miller (1953) represents the next contribution to the speculations about such system possibilities.

The first detailed system design and development of multiplexing systems based on other orthogonal functions was described by Ballard in a series of papers (1962a), (1962b), and (1962c) and (1963). In the first of these papers the word "orthomux" was coined to describe the multiplexing systems in which the message waveforms are linearly multiplied by the orthogonal functions in the transmitter and are recovered by correlation in the receiver. The general block diagram of an orthomux transmitter and receiver is shown in Figure 1. The orthogonal waveforms are normalized so that

$$\int_0^T 0_n(t)\,0_m(t)\,dt = \delta_{nm} \qquad (1)$$

where δ_{nm} is the Kronecker delta, which is one when the subscripts are the same and zero otherwise. The output of the multiplex system is the waveform:

$$f_m(t) = \sum_{i=1}^{N} m_i(t)\,0_i(t) \qquad (2)$$

It is necessary to assume that the orthogonality interval (0,T) is short enough so that the message waveforms vary little during the interval, otherwise the terms in the series would no longer be orthogonal to one another. The received waveform is also $f_m(t)$ if the channel is perfect. The j^{th} message waveform can then be recovered by multiplying the received waveform by $0_j(t)$ and integrating

[redacted]

The performance of the orthomux system when additive white Gaussian noise is introduced into the channel is shown in Tables 1 and 2. "Y" is the printout of the amplitude of the disturbing noise and FMT is the transmitted signal plus noise. The transmitted messages are equal to unity in each channel. MR1, MR2, and MR3 are the received signals while ERR1, ERR2, and ERR3 are the errors in the received messages at the corresponding times. This simulation also includes error due to time truncations so that at TIME=5.0 the error in each channel due to the influence of noise is small indeed.

It is possible to simulate multiplex systems based on analog or binary waveforms using DSL-90. In particular it is possible to use the switching functions incorporated in DSL-90 to form the standard Boolean logic functions such as the Nand, Nor, Exclusive-Or, etc. In this way any combinational or sequential binary system can be simulated on the digital computer before going to the expense of building hardware.

ANALOG SIMULATION

The digital computer simulation was followed by an analog computer simulation of the orthogonal multiplex system. The digital computer model of the multiplex system is a precise mathematical representation which yields a great deal of information in either numerical or graphical form. The analog computer model is more realistic since the system is essentially constructed from easily interconnected building blocks. These blocks are electronic circuits which perform the identical operations that the circuitry in a real system would perform. Because of this the analog computer yields information about the practical difficulties which would be encountered in constructing such a system and is thus the logical second step in a simulation effort. For example the analog simulation forced a detailed electronic system plan and drawing to be made which more nearly revealed the true complexity of the system, and it further brought to light the expense and difficulty that is encountered in constructing accurate high speed four quadrant multipliers.

The real exponential set was selected to investigate because the circuitry required to generate these functions is very simple. It is possible to generate a decaying exponential by impressing an impulse function on an RC network. The n^{th} order function of the set is formed by over the orthogonality interval:

$$\int_0^T \theta_j(t) f_m(t)\,dt = \int_0^T \theta_j(t) \sum_{i=1}^{N} m_i(t)\theta_i(t)\,dt =$$

$$\sum_{i=1}^{N} m_i(t) \int_0^T \theta_j(t)\theta_i(t)\,dt = m_j(t) \qquad (3)$$

The assumption that $m_j(t)$ is essentially constant over the orthogonality interval was used in removing it from under the integral sign. It is seen that the orthomux system is successful in recovering the original message waveforms, as long as the receiver is in time synchronization with the transmitter.

Double sideband modulation FDM and pulse amplitude modulation TDM are members of the orthomux class because the message waveforms are multiplied by orthogonal sinusoids and time pulses respectively in these systems. Other FDM and TDM systems are essentially orthomux systems except that instead of modulating the basic orthogonal function by multiplication, some other form of modulation (such as frequency modulation of a sine wave) is used which does not disturb the orthogonality of the $\theta_n(t)$.

Many sets of orthogonal functions are available for use in an orthomux system. Ballard designed systems using Legendre polynomials and orthogonal binary (Rademacher) functions. Karp and Higuchi (1963) analyzed modified Hermite polynomials, and Judge (1962) analyzed another set of binary functions. The availability of the many types of functions for use in orthomux systems led naturally to the question of which is optimum for a particular channel and its advantage, if any, over conventional systems.

Although the orthomux model is capable of producing an infinite variety of new multiplexing systems, it is not capable of describing all possible multiplexing systems. A simple multiplexing system for which no orthomux model can be derived is called amplitude division multiplexing or ADM. In this case the amplitude of a single waveform is determined by the value of all the input messages. Another multiplexing system for which no orthomux system can be derived is described by Titsworth (1963). The existence of these multiplexing systems which do not fit into the framework of the orthomux model raises another question: Does there exist a model which will describe all conceivable multiplexing systems? The "generalized orthomux" model of Figure 2 is capable of describing both ADM and Titsworth's system. By arguments similar to those in Wozencraft and Jacobs (1965), the generalized orthomux model can be shown to be capable of describing any system in which there are a finite number of messages. The model is an orthomux system preceded by a one-to-one vector transformation $\vec{M}$.

OPTIMUM MULTIPLEXING SYSTEMS

Both the transformation $\vec{M}$ and the type of orthonormal waveforms $0_n(t)$ must be considered for determination of which multiplexing system is optimum for a given channel. For a channel which disturbs the transmitted signal only by the addition of independent white Gaussian noise, it is possible to make definite statements about the optimum channel. The optimum transformation is the one that results in the possible output waveforms $f_m(t)$ being as far apart as possible in the vector space formed by taking each of the orthonormal $0_n(t)$ as a unit vector. However, this "simplex" arrangement of possible waveforms is not significantly superior to an orthogonal one if the number of possible waveforms is large (Wozencraft and Jacobs, 1965). Thus it may be said that a transformation that leads to the possible $f_m(t)$ being orthogonal is about as good as can be expected. It turns out that the type of orthonormal waveforms used as a basis is immaterial for this channel, and ease of implementation is the only important consideration in the selection (Shelton, 1967a).

For other channel models the $\vec{M}$ transformation described above is not necessarily the best. The ordinary orthomux system has a transformation that minimizes the effect of errors in the output of a correlator on other message outputs since each correlator determines only one message output. This is probably the wisest choice if the channel characteristics are not precisely known. For a channel which has a limited bandwidth available, the set of orthonormal waveforms having minimum bandwidth is a good choice. If the criterion of bandwidth is that frequency band outside of which none of the orthonormal functions has more than 1/12 of its energy, then the optimum set is the family of prolate spheroidal wave functions (Landau and Pollack, 1962). These functions are unattractive from the point of view of implementation. Certain FDM systems can achieve almost as small a bandwidth and are much more attractive from a practical standpoint. For a channel with a peak limitation, binary output signals are optimum, of course. Physical channels have a combination of constraints, which makes determination of the optimum system difficult. A more practical approach is to select several sets of orthonormal functions that are attractive from the inplementation viewpoint, and then to make tests of their performance in a simulation program. The complete multiplexing system and the appropriate channel can be simulated at an early stage of system design. The actual performance parameters can be determined by the simulation program at far less cost than system breadboarding.

In this paper the results of both digital and analog simulations of multiplexing system performance are presented. The results are mostly for the real exponential set of orthonormal functions which are attractive from the standpoint of equipment simplicity. The purpose of this paper, however, is primarily to illustrate the simulation approach to multiplexing system design and analysis and not to examine the real exponential set in detail.

DIGITAL SIMULATION

The simulation language chosen for this study was DSL-90 (Syn and Wyman, 1965). This language is a system based on Fortran IV. It is non-procedural in that it utilizes a sorting routine to develop the problem structure. The language is composed of a basic set of functional blocks to represent conventional analog components from which the model is constructed. Figure 3 and Figure 4 show the functional blocks with accompanying descriptions. In addition, DSL-90 contains a routine which performs automatic plotting of the system variables, using an X-Y plotter. DSL-90 is part of the IBM Share Library and instruction manuals and information are available from the IBM Corporation. It is only necessary to scan Figures 3 and 4 to realize the capability of the language.

Several criteria are useful in evaluating the performance of any multiplexing system. One important consideration is a measure of interchannel interference or crosstalk. In an orthomux system, distortion can be caused by errors in the orthogonality of the signals, by frequency truncation (filtering), amplitude limiting, synchronization errors and additive noise. Other considerations in the evaluation are determination of the peak-to-average power ratio and error in the received message due to various system imperfections and external influences.

The results of the simulation of an orthogonal multiplex system based on the orthonormal set of real exponential functions is presented in this paper to illustrate the procedure. To simulate a system based on another set of orthogonal functions requires only that the functions and system constants be changed. The majority of the program is unchanged. Figure 5 shows the first three functions of the real exponential set. These functions are orthonormal over the interval $(0,\infty)$. In a pulsed multiplex system it is necessary to truncate the signals in a finite time, and this causes time truncation crosstalk. Figure 6 shows message error versus truncation time, and from a graph such as this it is possible to select the minimum allowable truncation time for a specified maximum message error.

The effect of frequency limiting or filtering on the transmitted signal is shown in Figure 7. The signal is shown without filtering ($\omega=\infty$) and also after it has passed through a one pole filter with normalized bandwidth from $\omega=4$ to $\omega=1$. Distortion increases as the filter cutoff frequency becomes smaller. The resultant message error for each of the three channels of the system as a function of filter bandwidth is shown in Figure 8. The channel filter causes both amplitude distortion and delay of the transmitted signal. It is possible to compensate for the error due to the delay of the transmitted signal by delaying the operation of the receiver by an equal amount. This is termed synchronous operation and results in a great improvement in performance. Figure 10 shows the improvement, and for this case the optimum delay is t=0.05, which lowers the error for channels 2 and 3 from about 40% to 5%. This is an example of the principle that the receiver should be matched to the received wave shape and not to the transmitted wave shape.

Practical transmitters have peak-to-average power limitations, and it is advantageous to know the effect of clipping or amplitude limiting of the transmitted messages on the real exponential set. The error naturally increases as the clipping becomes more severe, as Figure 9 shows.

summing the correctly weighted outputs of the first n of the RC networks. Another method of generating the exponential set is shown in Figure 11. The results of the simulation were in agreement with the digital computer results in this instance. Several graphs are included to show the general nature of the analog computer outputs. Figure 12 shows time truncation error for various values of truncation time. Figure 13 shows the effect of frequency limiting on the transmitted signal for several values of filter cutoff frequency, and Figure 14 shows the total error on a per channel basis due to both time truncation and filtering for a specific filter cutoff frequency.

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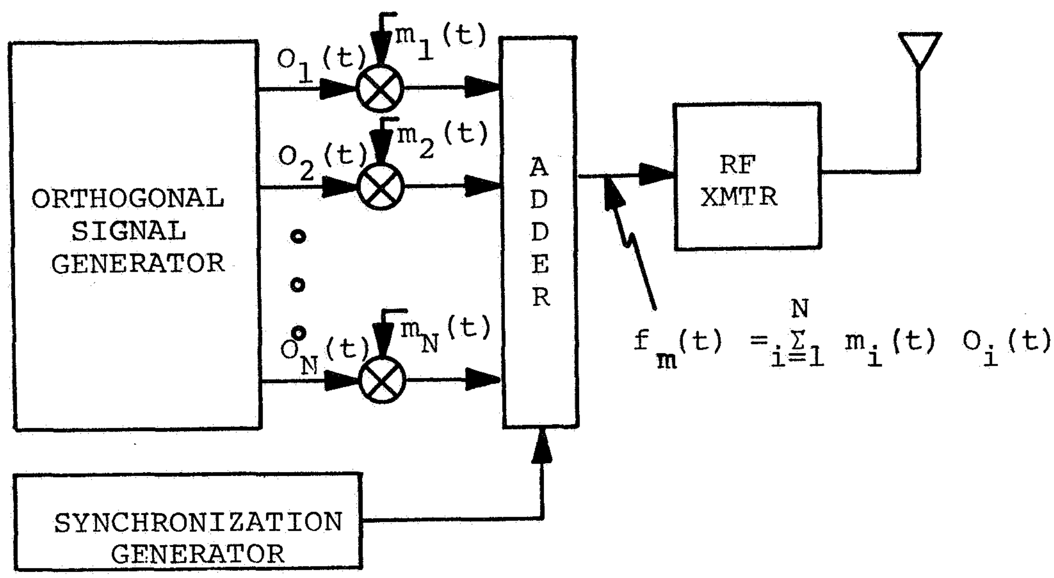
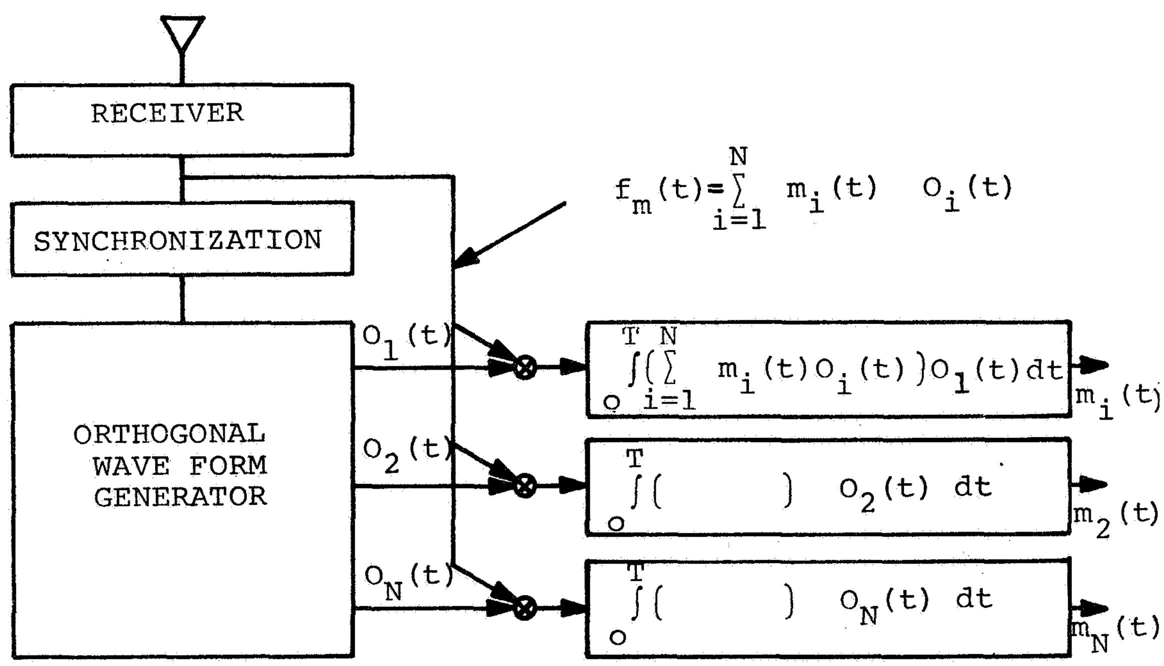


a) Transmitter Block Diagram



b) Receiver Block Diagram

Figure 1. Orthomux System a) Transmitter b) Receiver

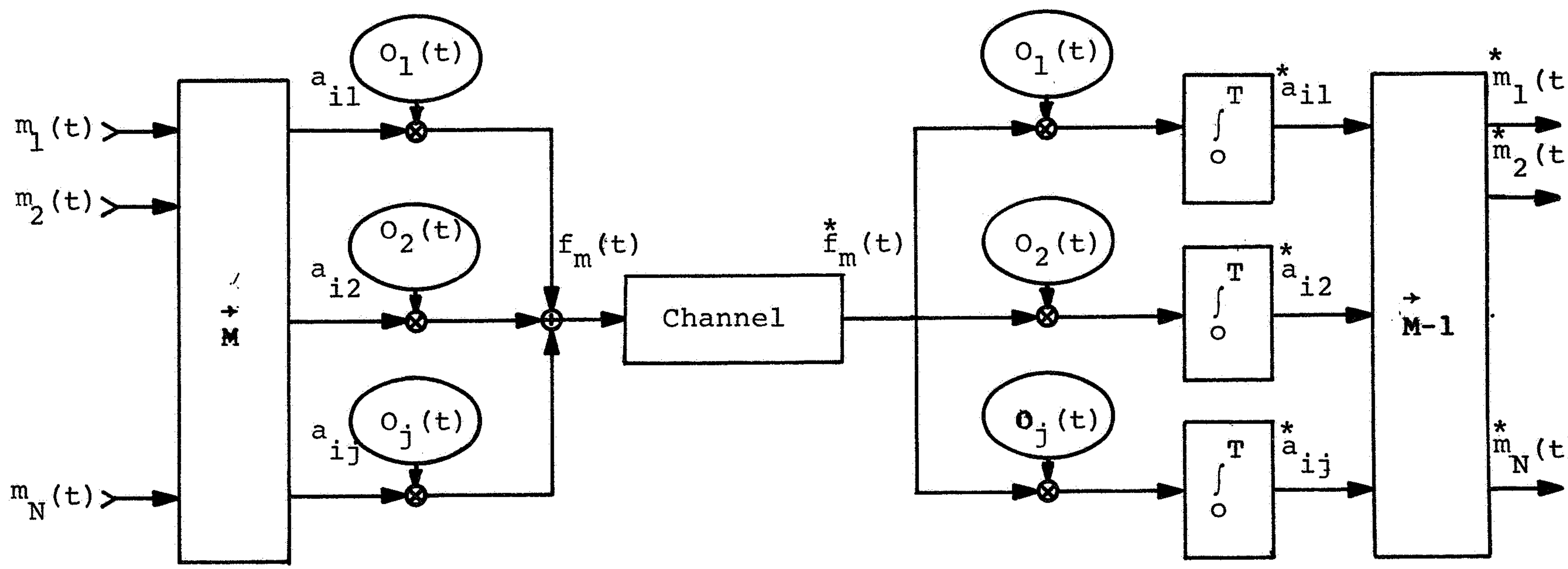


Figure 2. "Generalized Orthomux" Model.

Figure3 FUNCTIONAL DESCRIPTION OF STANDARD DSL/90 BLOCKS

	GENERAL FORM	FUNCTION
**	Y = INTGRL (IC, X) Y(0) = IC INTEGRATOR	$Y = \int_0^t X \, dt + IC$ EQUIVALENT LAPLACE TRANSFORM: $\frac{1}{S}$
*	Y = MODINT (IC, P_1, P_2, X) MODE-CONTROLLED INTEGRATOR	$Y = \int_0^t X \, dt + IC$ $P_1 = 1, P_2 = 0$ Y = IC $P_1 = 0, P_2 = 1$ Y = LAST OUTPUT $P_1 = 0, P_2 = 0$
*	Y = REALPL (IC, P, X) Y(0) = IC 1ST ORDER SYSTEM (REAL POLE)	$P\dot{Y} + Y = X$ EQUIVALENT LAPLACE TRANSFORM: $\frac{1}{PS+1}$
*	Y = LEDLAG (IC, P_1, P_2, X) Y(0) = IC LEAD-LAG	$P_2\dot{Y} + Y = P_1\dot{X} + X$ EQUIVALENT LAPLACE TRANSFORM: $\frac{P_1S+1}{P_2S+1}$
*	Y = CMPXPL (IC_1, IC_2, P_1, P_2, X) Y(0) = IC_1 $\dot{Y}(0) = IC_2$ 2ND ORDER SYSTEM (COMPLEX POLE)	$\ddot{Y} + 2P_1P_2\dot{Y} + P_2^2Y = X$ EQUIVALENT LAPLACE TRANSFORM: $\frac{1}{S^2+2P_1P_2S+P_2^2}$
	Y = DERIV (IC, X) Y(0) = IC DERIVATIVE	$Y = \frac{dX}{dt}$ QUADRATIC INTERPOLATION EQUIVALENT LAPLACE TRANSFORM: S
	Y = DELAY (N, P, X) P = TOTAL DELAY IN TERMS OF INDEPENDENT VAR. N = MAX NO. OF POINTS DELAY DEAD TIME (DELAY)	$Y(t) = X(t-P)$ $t \geq P$ $Y = 0$ $t < P$ EQUIVALENT LAPLACE TRANSFORM: e^{-PS}
	Y = ZHOLD (P, X) Y(0) = 0 ZERO-ORDER HOLD	Y = X P = 1 Y = LAST OUTPUT P = 0 EQUIVALENT LAPLACE TRANSFORM: $\frac{1}{S}(1-e^{-st})$
	Y = IMPL (IC, ERROR, FUNCT) IMPLICIT FUNCTION	Y = IC $t = 0$ FIRST ENTRY Y = FUNCT(Y) $t \geq 0$ $\lvert Y - FUNCT(Y) \rvert \leq ERROR \cdot \lvert Y \rvert$

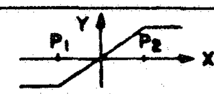
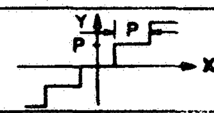
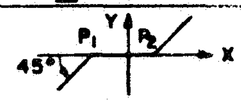
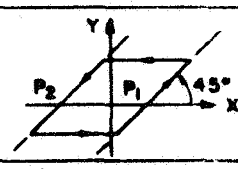
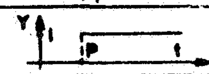
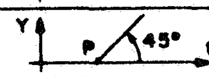
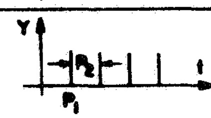
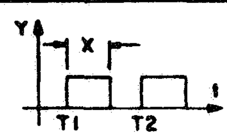
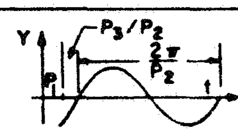
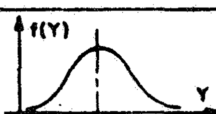
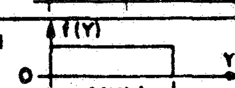
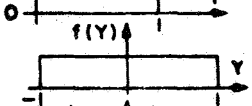
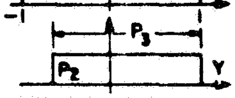
SWITCHING FUNCTIONS

Y = FCNSW (P, X_1, X_2, X_3) FUNCTION SWITCH	$Y = X_1$ $P < 0$ $Y = X_2$ $P = 0$ $Y = X_3$ $P > 0$
Y = INSW (P, X_1, X_2) INPUT SWITCH (RELAY)	$Y = X_1$ $P < 0$ $Y = X_2$ $P \geq 0$
Y_1, Y_2 = OUTSW (P, X) OUTPUT SWITCH	$Y_1 = X, Y_2 = 0$ $P < 0$ $Y_1 = 0, Y_2 = X$ $P \geq 0$
Y = COMPAR (X_1, X_2) COMPARATOR	$Y = 0$ $X_1 < X_2$ $Y = 1$ $X_1 \geq X_2$
Y = RST (P_1, P_2, P_3) RST FLIP-FLOP	$Y = 0$ $P_1 > 0$ $Y = 1$ $P_2 > 0$, ($P_1 \leq 0$) $Y = 0$ $P_3 > 0$, $Y_{n-1} = 1$, ($P_2 \leq 0$, $P_1 \leq 0$) $Y = 1$ $P_3 > 0$, $Y_{n-1} = 0$, " "

* THESE FOUR BLOCKS EXIST AS BUILT-IN MACROS WITHIN DSL. IN-LINE CODE REPRESENTING AN EQUIVALENT INTEGRATOR CIRCUIT IS GENERATED FOR EACH USE TO PERMIT THE USE OF CENTRALIZED INTEGRATION SCHEMES WITHIN THE BLOCKS.

** INTGRL MUST BE THE RIGHTMOST TERM FOR EACH LEVEL OF USAGE. IF X IS A SINGLE VARIABLE NAME THEN IT MUST BE UNIQUE WITHIN THE PROBLEM. IC MUST ALSO BE UNIQUE. (-IC IS NOT VALID). A LITERAL MAY BE USED FOR IC. ALSO SEE SECT. 5-1.

Figure 4 FUNCTION GENERATORS

GENERAL FORM	FUNCTION
Y=AFGEN (FUNCT, X) ARBITRARY LINEAR FUNCTION GENERATOR	Y=FUNCT (X) $X_0 \le X \le X_n$ LINEAR INTERPOLATION Y=FUNCT (X_0) $X < X_0$ Y=FUNCT (X_n) $X > X_n$
Y=NLFGEN (FUNCT, X) NON-LINEAR FUNCTION GENERATOR	Y=FUNCT (X) $X_0 \le X \le X_n$ QUADRATIC INTERPOLATION (LA GRANGE) Y=FUNCT (X_0) $X < X_0$ Y=FUNCT (X_n) $X > X_n$
Y=LIMIT (P_1, P_2, X) LIMITER	$Y = P_1$ $X < P_1$ $Y = P_2$ $X > P_2$ $Y = X$ $P_1 \le X \le P_2$ 
Y=QNTZR (P, X) QUANTIZER	$Y = kP$ $(k-1/2)P < X \le (k+1/2)P$ $k = 0, \pm 1, \pm 2, \pm 3, \ldots$ 
Y=DEADSP (P_1, P_2, X) DEAD SPACE	$Y = 0$ $P_1 \le X \le P_2$ $Y = X - P_2$ $X > P_2$ $Y = X - P_1$ $X < P_1$ 
Y=HSTRSS (IC, P_1, P_2, X) Y(0) = IC HYSTERESIS LOOP	$Y = X - P_1$ $(X - X_{n-1}) > 0$ AND $Y_{n-1} \le (X - P_1)$ $Y = X - P_2$ $(X - X_{n-1}) < 0$ AND $Y_{n-1} \ge (X - P_2)$ OTHERWISE Y=LAST OUTPUT 
Y=STEP (P) STEP FUNCTION	$Y = 0$ $t < P$ $Y = 1$ $t \ge P$ 
Y=RAMP (P) RAMP FUNCTION	$Y = 0$ $t < P$ $Y = t - P$ $t \ge P$ 
Y=IMPULS (P_1, P_2) IMPULSE GENERATOR	$Y = 0$ $t < P_1$ $Y = 1$ $(t - P_1) = kP_2$ $Y = 0$ $(t - P_1) \ne kP_2$ $k = 0, 1, 2, 3, \ldots$ 
Y=PULSE (P, X) PULSE GENERATOR WITH P AS TRIGGER	$Y = 0$ INITIAL $Y = 1$ $T_k \le t < (T_k + X)$ $Y = 0$ OTHERWISE $k = 1, 2, 3, \ldots$ T_k = t OF PULSE k, P_k 
Y=SINE (P_1, P_2, P_3) P_2=FREQUENCY IN RADIANS/SEC. P_3=PHASE SHIFT IN RADIANS TRIGONOMETRIC SINE WAVE WITH AMPLITUDE, PHASE, AND DELAY	$Y = 0$ $t < P_1$ $Y = \mathrm{SIN}\,[P_2 \cdot (t - P_1) + P_3]$ $t \ge P_1$ 
Y=NORMAL (P_1, P_2, P_3) NOISE GENERATOR (NORMAL DISTRIBUTION)	Y=GAUSSIAN DISTRIBUTION WITH MEAN, P_2, AND STANDARD DEVIATION, P_3 (P_1=ANY ODD INTEGER) 
Y=UNZRPI (P_1) Y=UNMIPI (P_1) Y=UNATOB (P_1, P_2, P_3) NOISE GENERATOR (UNIFORM DISTRIBUTION)	Y=UNIFORM DISTRIBUTION 0 TO 1 (P_1=ANY ODD INTEGER)  Y=UNIFORM DISTRIBUTION, -1 TO +1  Y=UNIFORM DISTRIBUTION, P_2 TO $P_2 + P_3$ 

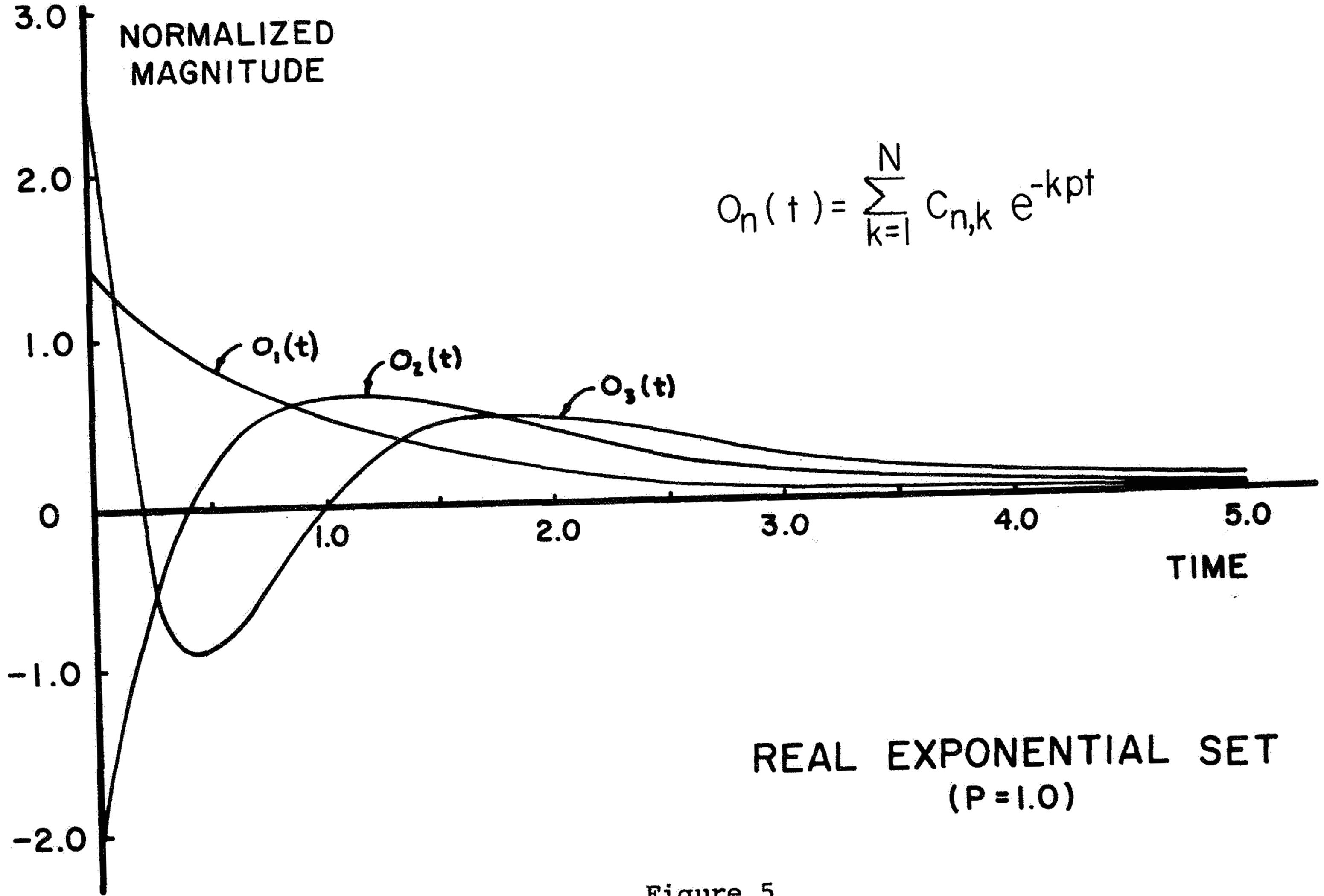

3.0
2.0
1.0
0
-1.0
-2.0
NORMALIZED MAGNITUDE
$O_n(t) = \sum_{k=1}^{N} C_{n,k}\, e^{-kpt}$
$O_1(t)$
$O_2(t)$
$O_3(t)$
1.0
2.0
3.0
4.0
5.0
TIME
REAL EXPONENTIAL SET
(P = 1.0)

Figure 5

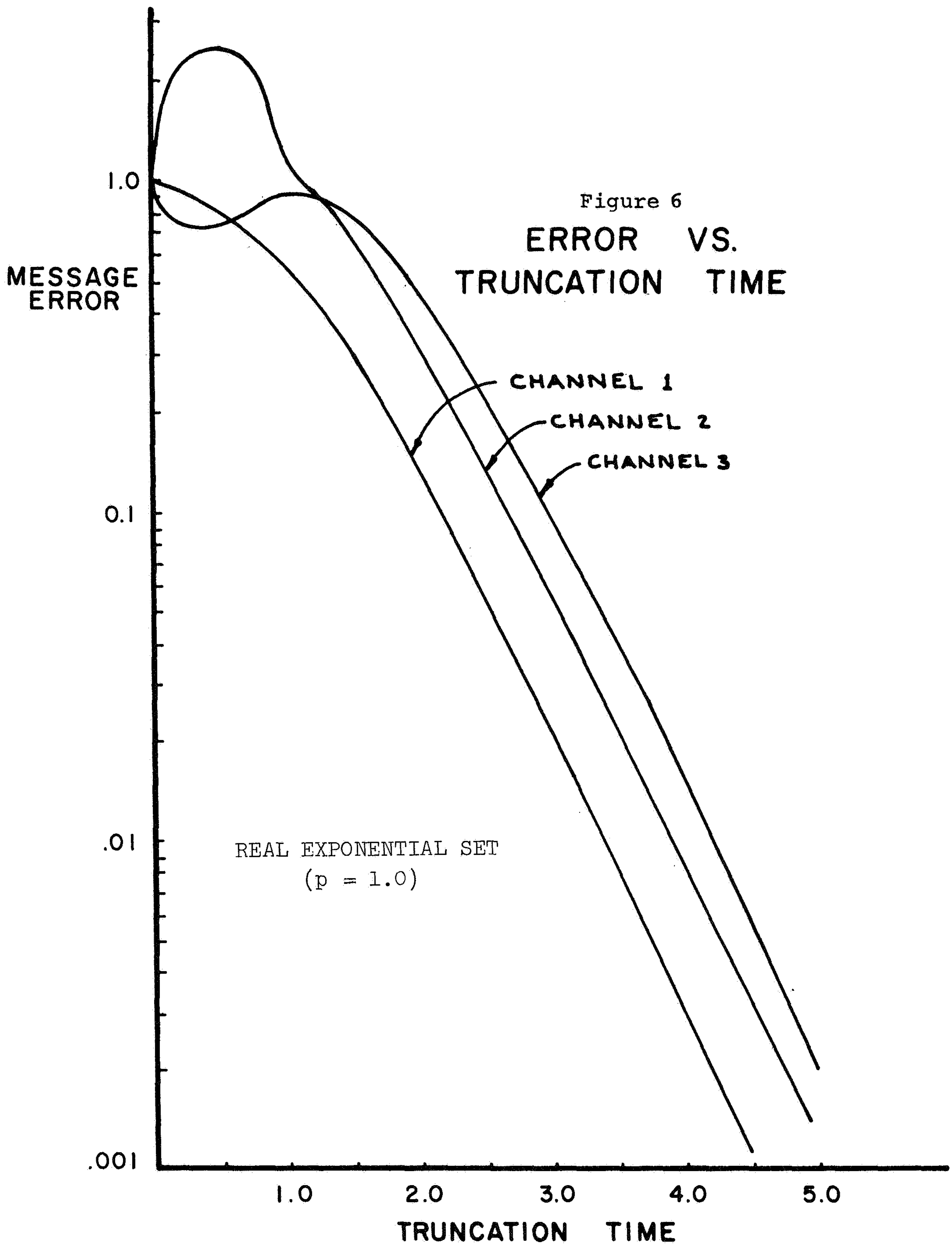


Figure 6
ERROR VS. TRUNCATION TIME
MESSAGE ERROR
1.0
0.1
.01
.001
CHANNEL 1
CHANNEL 2
CHANNEL 3
REAL EXPONENTIAL SET
(p = 1.0)
1.0
2.0
3.0
4.0
5.0
TRUNCATION TIME

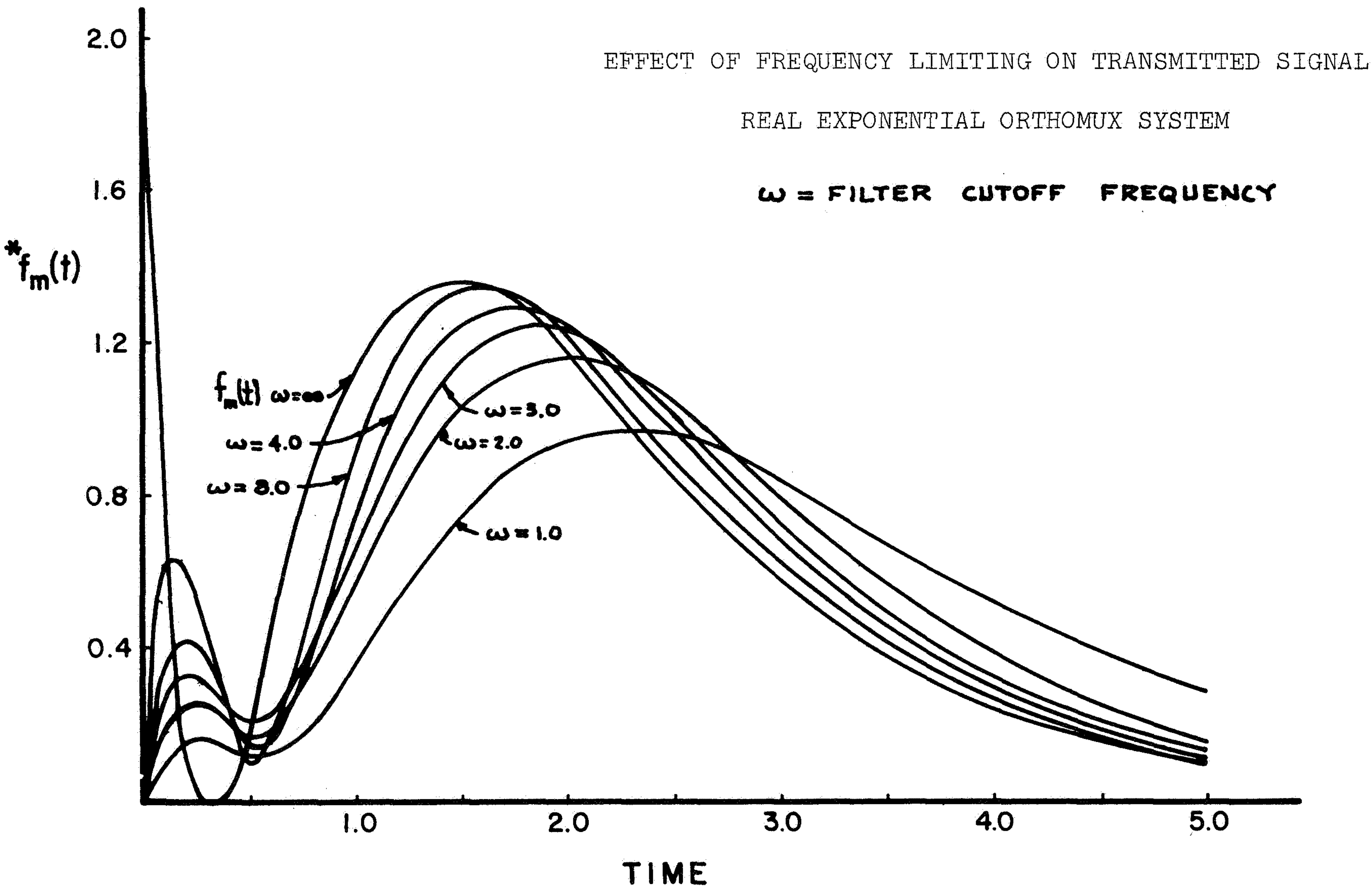

EFFECT OF FREQUENCY LIMITING ON TRANSMITTED SIGNAL
REAL EXPONENTIAL ORTHOMUX SYSTEM
ω = FILTER CUTOFF FREQUENCY
*f_m(t)
2.0
1.6
1.2
0.8
0.4
f_m(t) ω=∞
ω=4.0
ω=8.0
ω=3.0
ω=2.0
ω=1.0
1.0
2.0
3.0
4.0
5.0
TIME

Figure 7

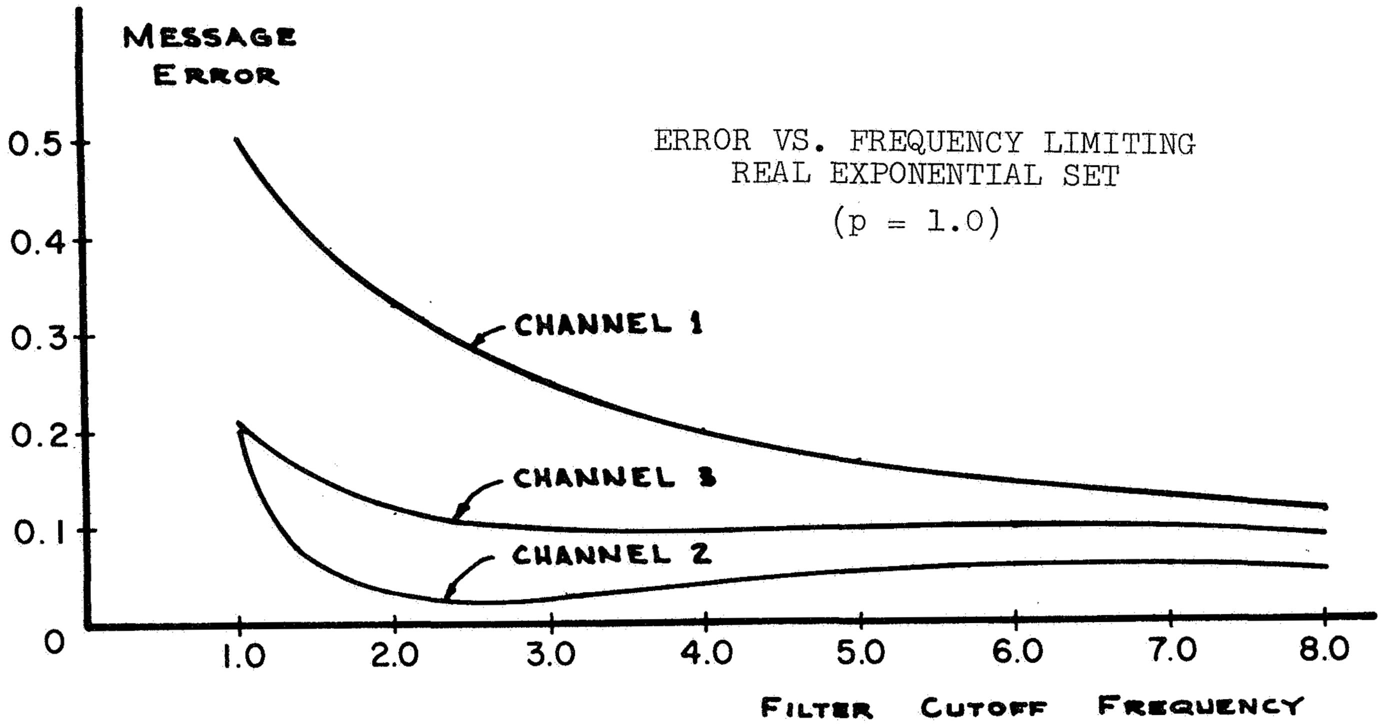


Figure 8

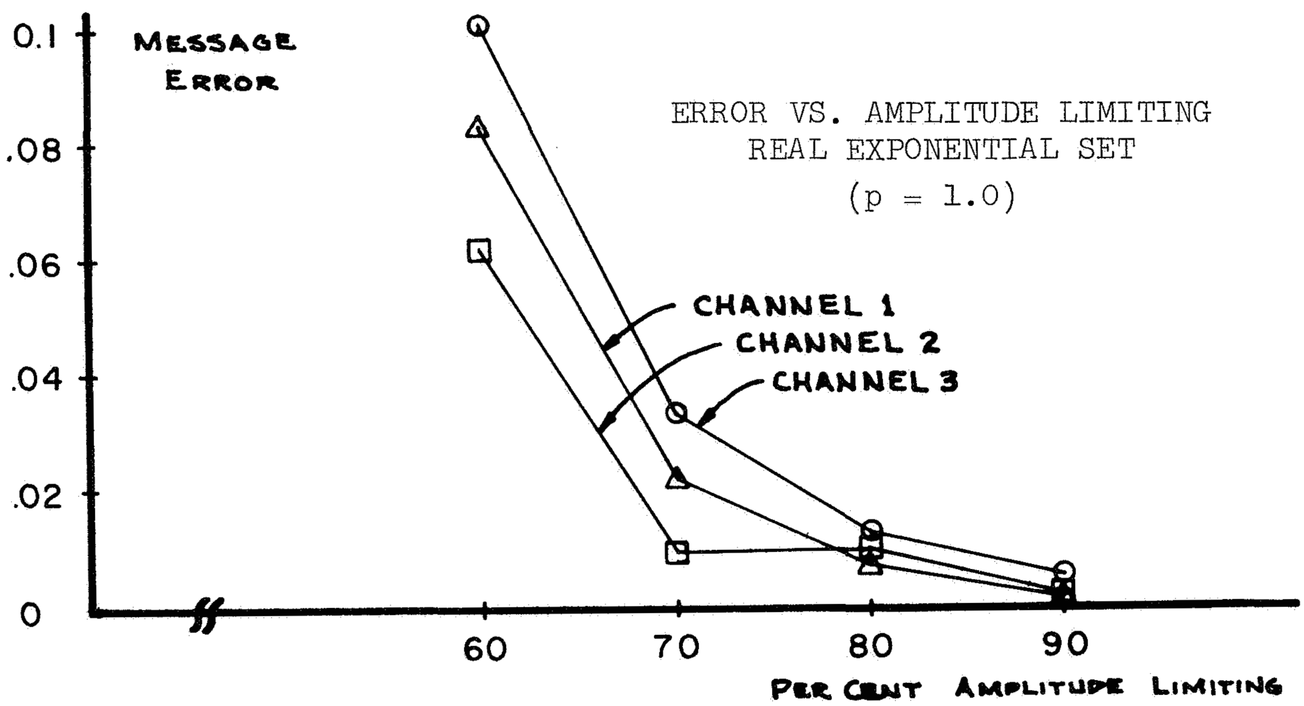


Figure 9

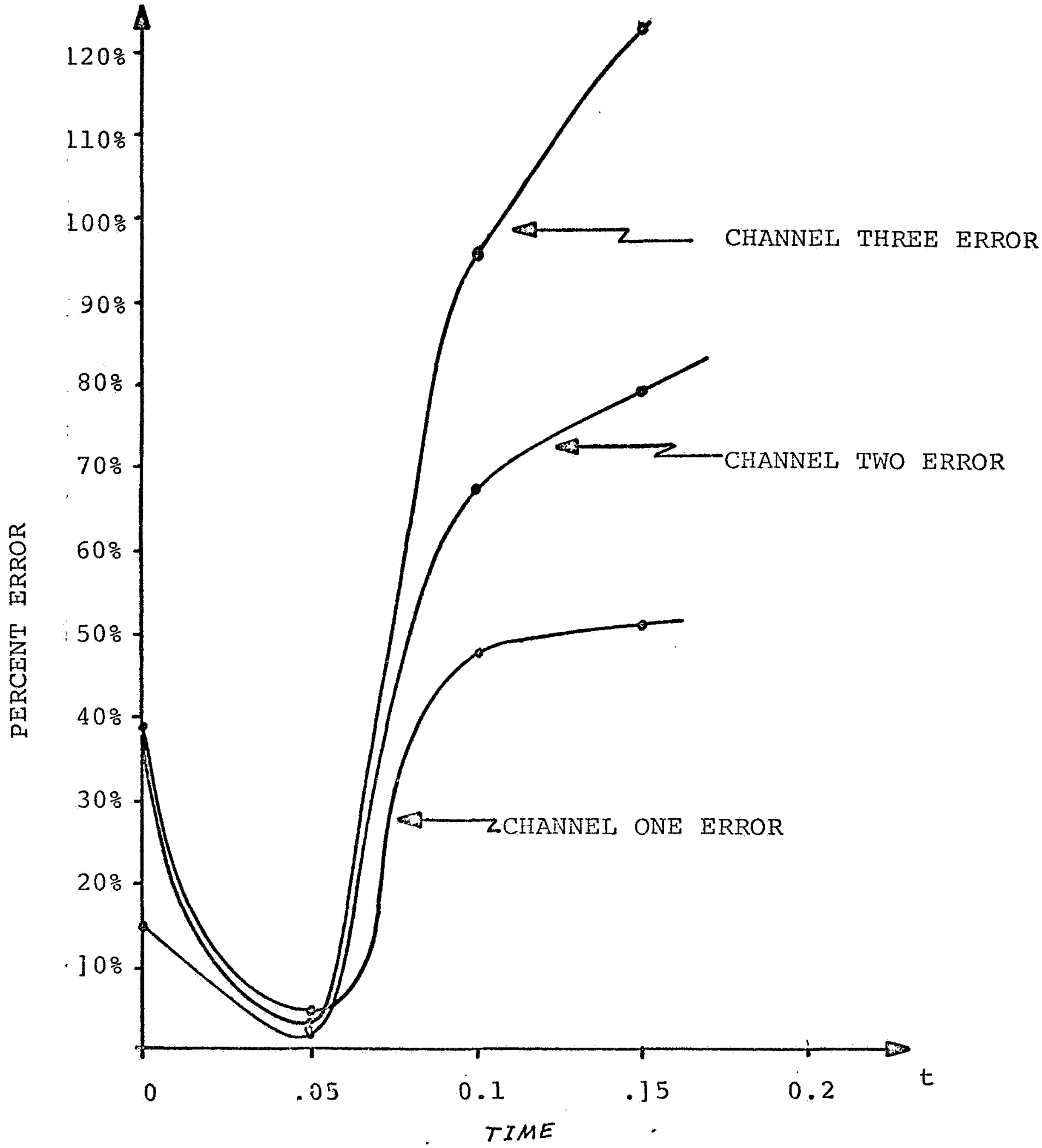


FIGURE 10 . Error due to the delay caused by the channel.

Table 1

SAME SYSTEM8 P1=15,P3=1.095,S/N=1/2

TIME	Y	FMT	MR1	MR2
0.	1.2854E 00	3.1491E 00	0.	0.
1.000E-01	-6.3669E-01	7.9668E-02	1.6408E-01	-2.0370E-01
2.000E-01	5.6079E-01	7.2784E-01	2.0898E-01	-2.4289E-01
3.000E-01	-2.7088E-01	-2.8175E-01	2.2005E-01	-2.4832E-01
4.000E-01	-1.0273E 00	-9.9805E-01	2.1455E-01	-2.4732E-01
5.000E-01	7.8073E-01	9.6655E-01	2.2983E-01	-2.4540E-01
6.000E-01	-2.2278E-01	1.7007E-01	2.4685E-01	-2.3911E-01
7.000E-01	-7.0295E-01	-9.3663E-02	2.8061E-01	-2.1790E-01
8.000E-01	8.6686E-01	1.6777E 00	3.2577E-01	-1.8070E-01
9.000E-01	-3.7580E-01	6.0874E-01	3.7625E-01	-1.2922E-01
1.000E 00	2.7256E 00	3.8502E 00	4.3402E-01	-6.0772E-02
1.100E 00	-1.6565E 00	-4.2648E-01	4.9762E-01	2.4581E-02
1.200E 00	-7.1738E-01	5.8510E-01	5.5598E-01	1.1122E-01
1.300E 00	-1.1489E 00	1.9629E-01	6.0823E-01	1.9547E-01
1.400E 00	2.8661E-01	1.6489E 00	6.5632E-01	2.7862E-01
1.500E 00	8.3526E-01	2.1930E 00	7.0147E-01	3.6130E-01
1.600E 00	-1.3021E 00	3.3466E-02	7.4047E-01	4.3644E-01
1.700E 00	-1.0216E 00	2.7806E-01	7.7701E-01	5.0996E-01
1.800E 00	-1.6721E-01	1.0860E 00	8.0970E-01	5.7832E-01
1.900E 00	-6.6167E-02	1.1329E 00	8.3589E-01	6.3491E-01
2.000E 00	3.2698E-01	1.4667E 00	8.6049E-01	6.8962E-01
2.100E 00	1.2029E 00	2.2800E 00	8.8221E-01	7.3918E-01
2.200E 00	4.2293E-01	1.4359E 00	9.0052E-01	7.8190E-01
2.300E 00	4.7558E-01	1.4241E 00	9.1515E-01	8.1673E-01
2.400E 00	-1.2956E 00	-4.1079E-01	9.2789E-01	8.4761E-01
2.500E 00	4.3893E-01	1.2616E 00	9.3898E-01	8.7492E-01
2.600E 00	-1.9192E 00	-1.1565E 00	9.4867E-01	8.9914E-01
2.700E 00	-7.3677E-01	-3.1467E-02	9.5553E-01	9.1648E-01
2.800E 00	-3.6645E-02	6.1408E-01	9.6169E-01	9.3223E-01
2.900E 00	-5.9415E-01	5.0166E-03	9.6643E-01	9.4450E-01
3.000E 00	1.2598E 00	1.8105E 00	9.7107E-01	9.5660E-01
3.100E 00	-1.6199E 00	-1.1145E 00	9.7517E-01	9.6738E-01
3.200E 00	-4.2469E-01	3.8396E-02	9.7882E-01	9.7703E-01
3.300E 00	-2.0741E 00	-1.6503E 00	9.8105E-01	9.8299E-01
3.400E 00	-9.2060E-01	-5.3317E-01	9.8318E-01	9.8870E-01
3.500E 00	-5.4092E-01	-1.8712E-01	9.8444E-01	9.9208E-01
3.600E 00	3.4550E-01	6.6830E-01	9.8635E-01	9.9726E-01
3.700E 00	5.5136E-02	3.4941E-01	9.8713E-01	9.9938E-01
3.800E 00	2.1478E 00	2.4159E 00	9.8816E-01	1.0022E 00
3.900E 00	-1.7496E 00	-1.5056E 00	9.8898E-01	1.0044E 00
4.000E 00	-9.1035E-01	-6.8831E-01	9.8911E-01	1.0048E 00
4.100E 00	1.1514E-01	3.1705E-01	9.8966E-01	1.0063E 00
4.200E 00	-3.7928E-02	1.4558E-01	9.8967E-01	1.0064E 00
4.300E 00	6.8394E-01	8.5066E-01	9.9033E-01	1.0082E 00
4.400E 00	-2.1828E 00	-2.0314E 00	9.9053E-01	1.0087E 00
4.500E 00	2.1834E 00	2.3209E 00	9.9062E-01	1.0090E 00
4.600E 00	4.2120E-01	5.4594E-01	9.9052E-01	1.0087E 00
4.700E 00	2.2553E-01	3.3870E-01	9.9052E-01	1.0087E 00
4.800E 00	-3.7002E-01	-2.6737E-01	9.9071E-01	1.0093E 00
4.900E 00	-1.2549E-02	8.0537E-02	9.9093E-01	1.0099E 00
5.000E 00	5.8809E-01	6.7248E-01	9.9080E-01	1.0095E 00

VARIABLE	MINIMUM	MAXIMUM
ØRF1	9.5288E-03	1.4142E 00
ØRF2	-2.0000E 00	6.6667E-01

Table 2

MR3	ERR1	ERR2	ERR3
0.	-1.0000E 00	-1.0000E 00	-1.0000E 00
1.9848E-01	-8.3592E-01	-1.2037E 00	-8.0152E-01
2.0988E-01	-7.9102E-01	-1.2429E 00	-7.9012E-01
2.0491E-01	-7.7995E-01	-1.2483E 00	-7.9509E-01
2.0944E-01	-7.8545E-01	-1.2473E 00	-7.9056E-01
1.9401E-01	-7.7017E-01	-1.2454E 00	-8.0599E-01
1.7653E-01	-7.5315E-01	-1.2391E 00	-8.2347E-01
1.4544E-01	-7.1939E-01	-1.2179E 00	-8.5456E-01
1.1134E-01	-6.7423E-01	-1.1807E 00	-8.8866E-01
8.5371E-02	-6.2375E-01	-1.1292E 00	-9.1463E-01
7.0683E-02	-5.6598E-01	-1.0608E 00	-9.2932E-01
7.3339E-02	-5.0238E-01	-9.7542E-01	-9.2666E-01
9.3852E-02	-4.4402E-01	-8.8878E-01	-9.0615E-01
1.2848E-01	-3.9177E-01	-8.0453E-01	-8.7152E-01
1.7528E-01	-3.4368E-01	-7.2138E-01	-8.2472E-01
2.3254E-01	-2.9853E-01	-6.3870E-01	-7.6746E-01
2.9340E-01	-2.5953E-01	-5.6356E-01	-7.0660E-01
3.6059E-01	-2.2299E-01	-4.9004E-01	-6.3941E-01
4.2945E-01	-1.9030E-01	-4.2168E-01	-5.7055E-01
4.9110E-01	-1.6411E-01	-3.6509E-01	-5.0890E-01
5.5470E-01	-1.3951E-01	-3.1038E-01	-4.4530E-01
6.1565E-01	-1.1779E-01	-2.6082E-01	-3.8435E-01
6.7064E-01	-9.9477E-02	-2.1810E-01	-3.2936E-01
7.1743E-01	-8.4854E-02	-1.8327E-01	-2.8257E-01
7.6036E-01	-7.2112E-02	-1.5239E-01	-2.3964E-01
7.9952E-01	-6.1023E-02	-1.2508E-01	-2.0048E-01
8.3517E-01	-5.1326E-02	-1.0086E-01	-1.6483E-01
8.6131E-01	-4.4466E-02	-8.3516E-02	-1.3869E-01
8.8558E-01	-3.8312E-02	-6.7770E-02	-1.1442E-01
9.0481E-01	-3.3565E-02	-5.5503E-02	-9.5187E-02
9.2413E-01	-2.8925E-02	-4.3400E-02	-7.5874E-02
9.4159E-01	-2.4826E-02	-3.2617E-02	-5.8412E-02
9.5739E-01	-2.1181E-02	-2.2968E-02	-4.2614E-02
9.6728E-01	-1.8946E-02	-1.7007E-02	-3.2723E-02
9.7684E-01	-1.6818E-02	-1.1298E-02	-2.3155E-02
9.8256E-01	-1.5564E-02	-7.9170E-03	-1.7442E-02
9.9138E-01	-1.3654E-02	-2.7440E-03	-8.6218E-03
9.9503E-01	-1.2873E-02	-6.1684E-04	-4.9695E-03
9.9993E-01	-1.1837E-02	2.2135E-03	-7.4811E-05
1.0038E 00	-1.1025E-02	4.4390E-03	3.7982E-03
1.0044E 00	-1.0892E-02	4.8050E-03	4.4410E-03
1.0071E 00	-1.0335E-02	6.3411E-03	7.1410E-03
1.0072E 00	-1.0328E-02	6.3638E-03	7.1835E-03
1.0104E 00	-9.6717E-03	8.1810E-03	1.0404E-02
1.0114E 00	-9.4706E-03	8.7404E-03	1.1399E-02
1.0119E 00	-9.3754E-03	9.0064E-03	1.1876E-02
1.0114E 00	-9.4757E-03	8.7287E-03	1.1381E-02
1.0114E 00	-9.4770E-03	8.7262E-03	1.1378E-02
1.0123E 00	-9.2887E-03	9.2531E-03	1.2327E-02
1.0134E 00	-9.0698E-03	9.8661E-03	1.3432E-02
1.0128E 00	-9.1974E-03	9.5097E-03	1.2789E-02

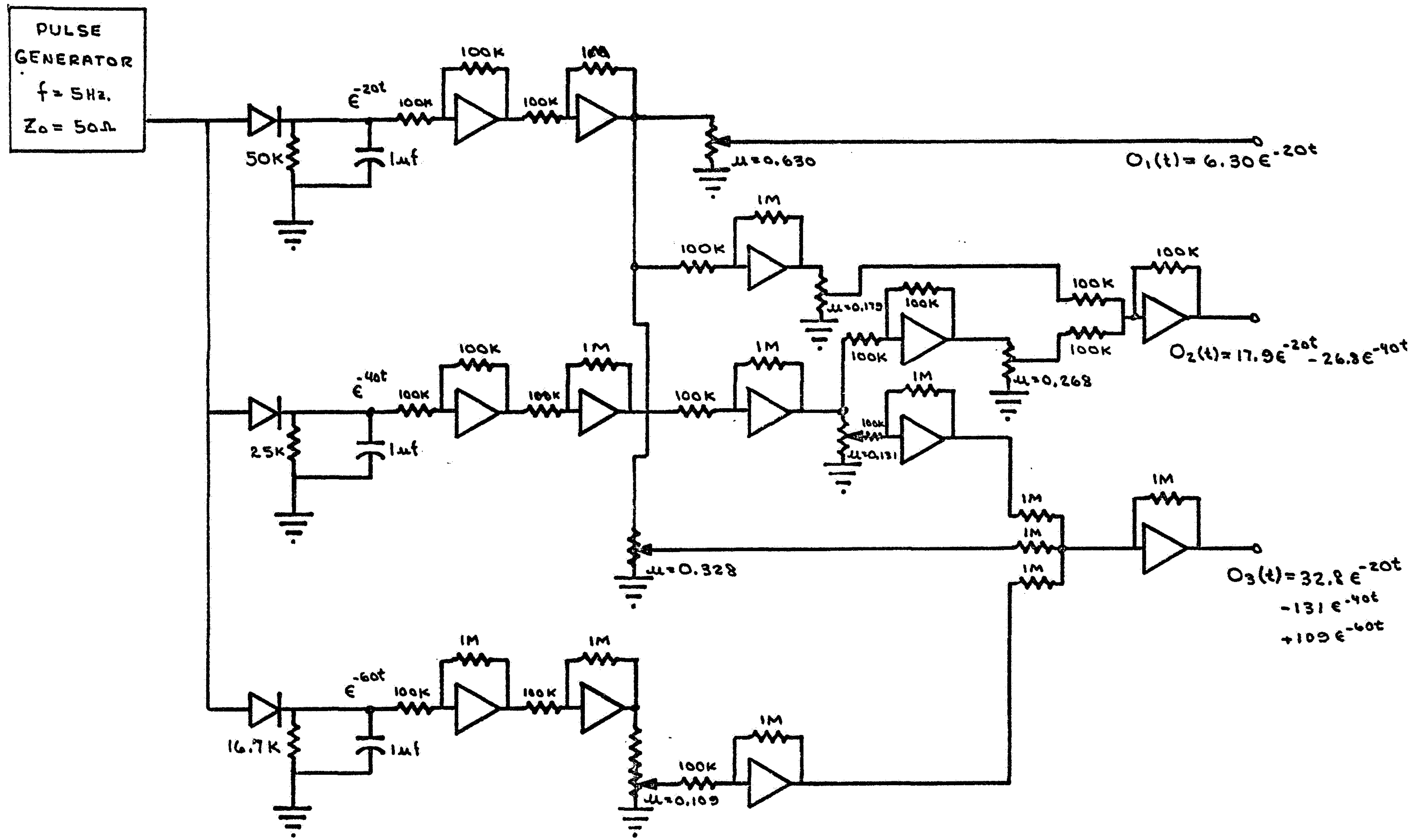


FIGURE 11 R-C NETWORKS, SUMMERS, AND AMPLIFIERS

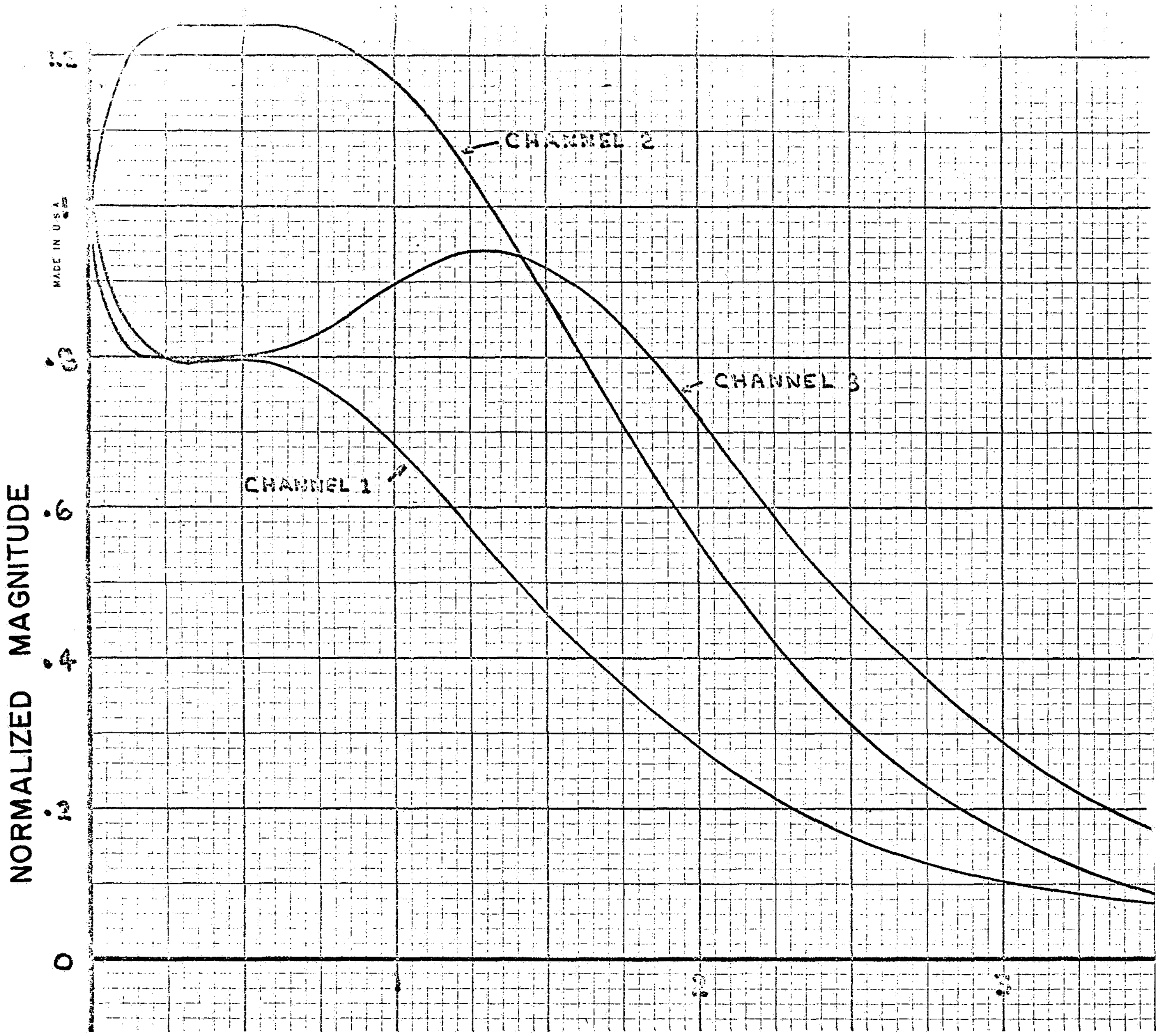
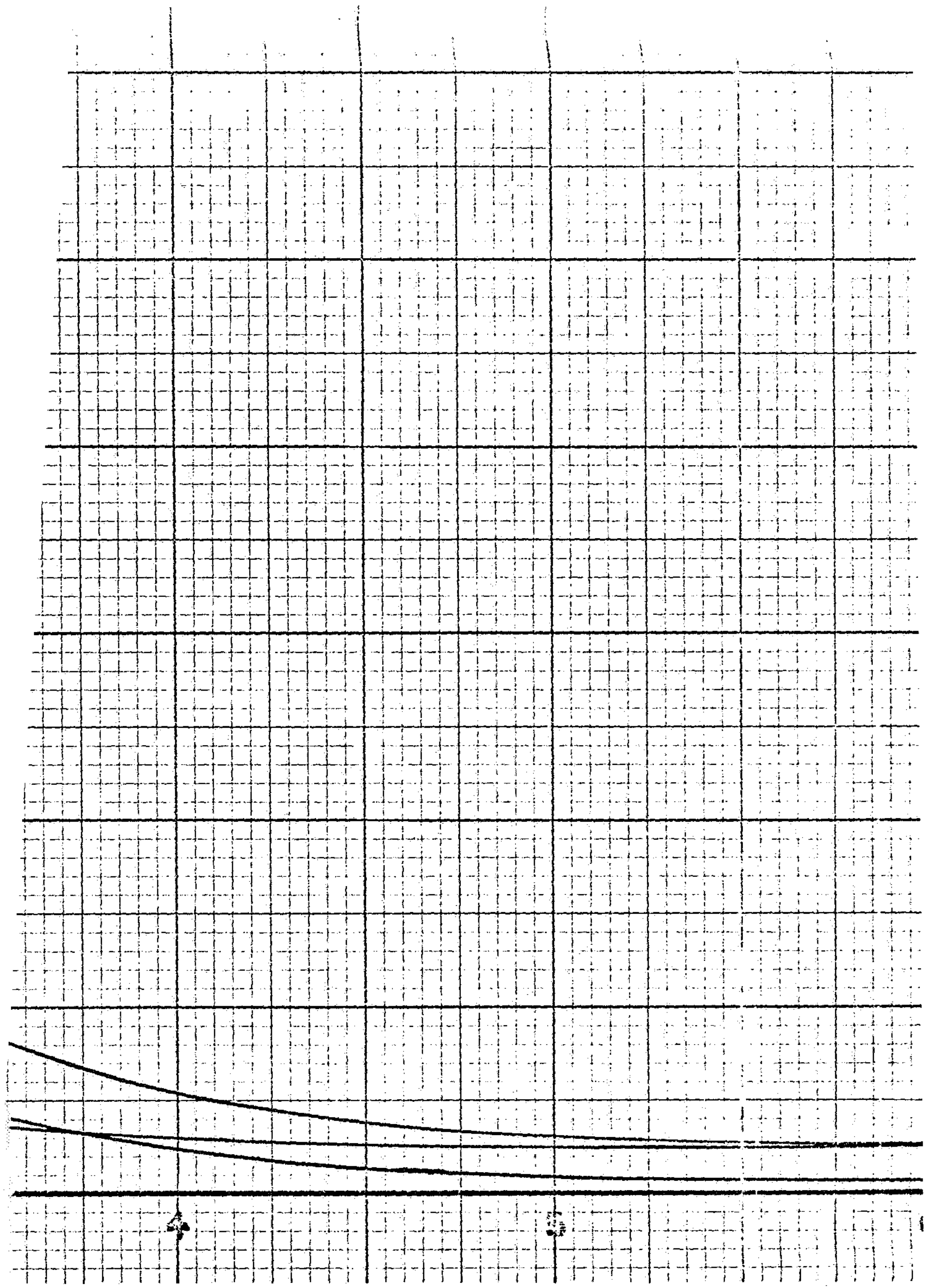


FIGURE 12 TIME TRUNCATION ERROR



S PT

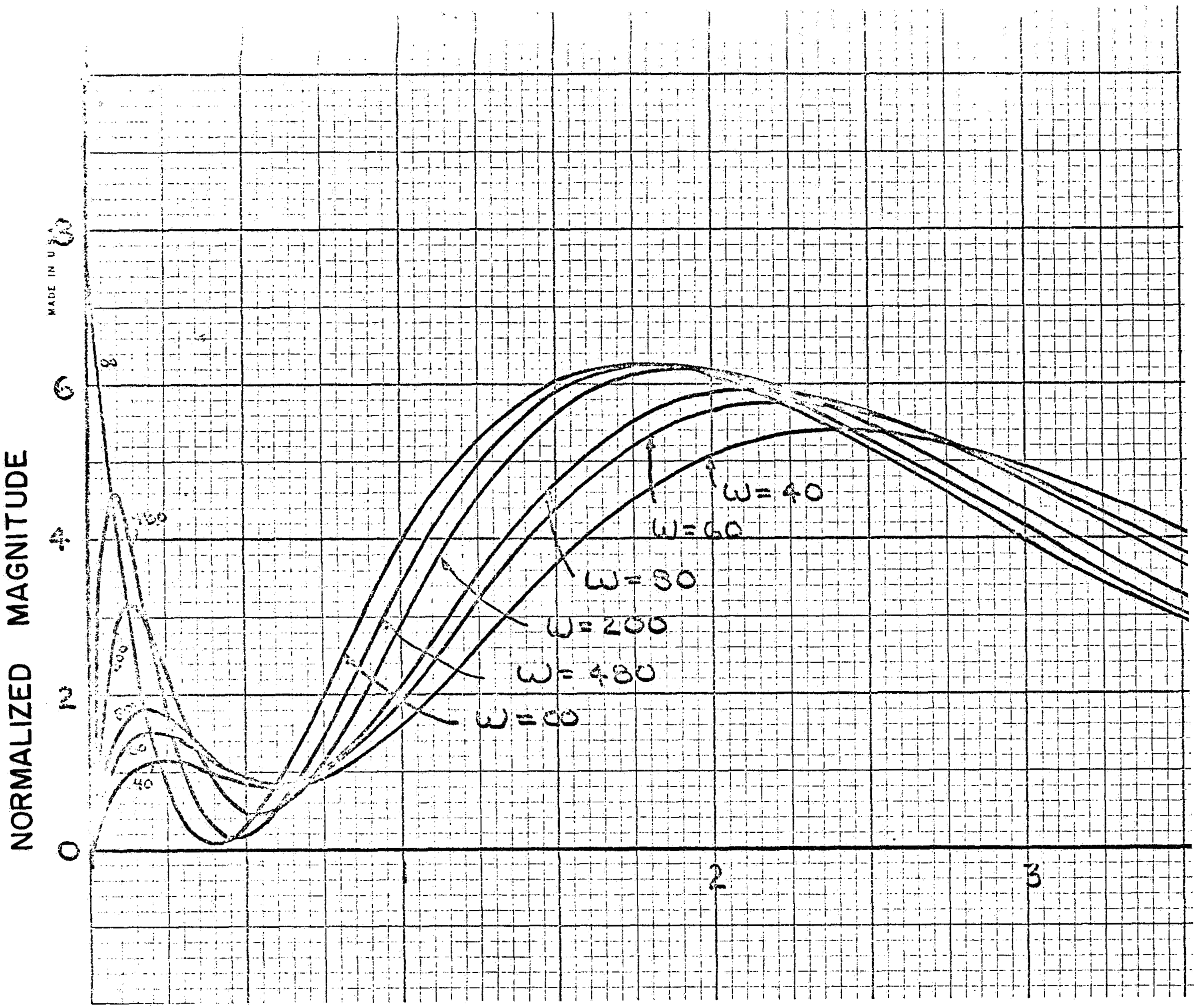
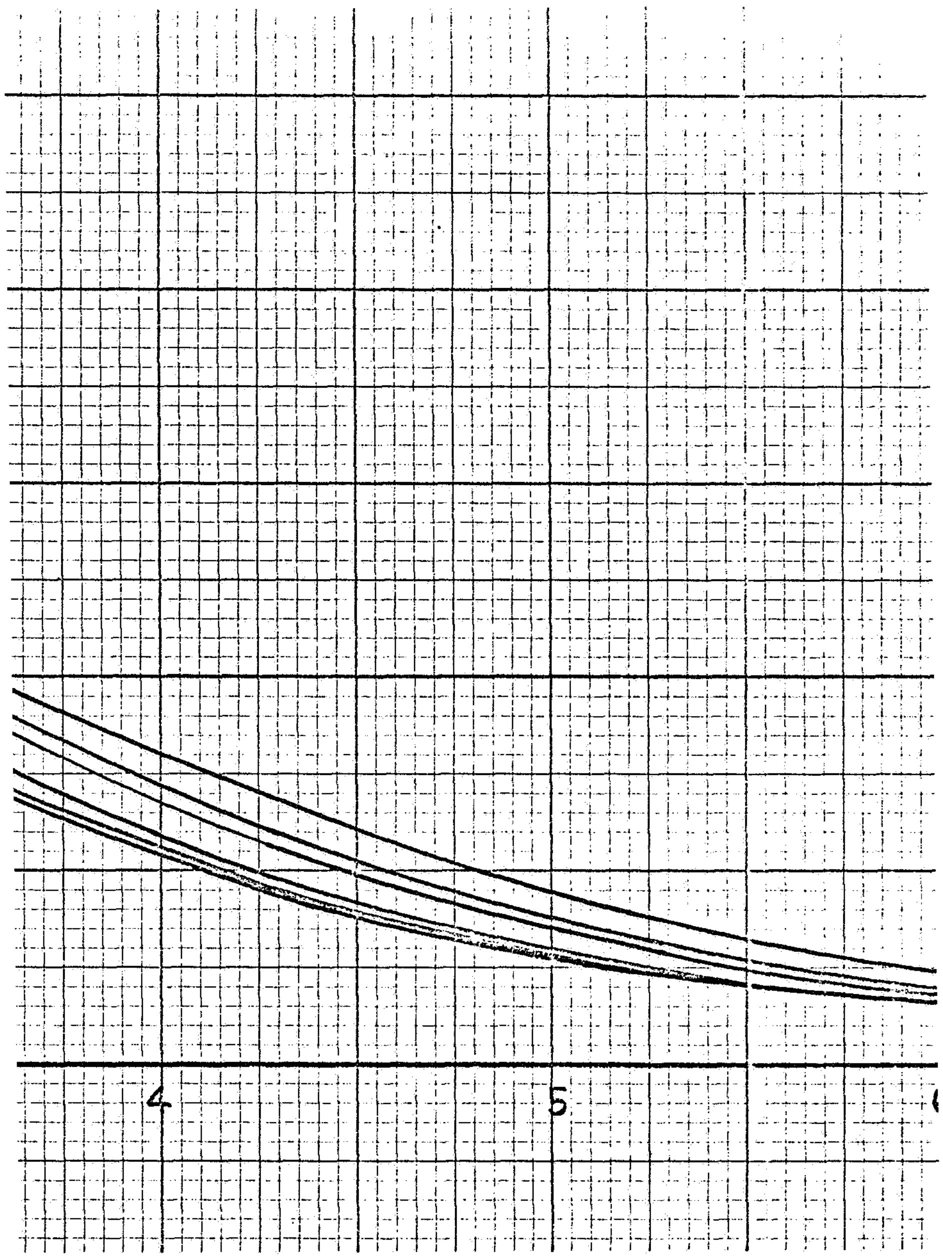


FIGURE 13 EFFECT OF FREQUENCY LIMITING ON T

SIGNAL (ω = FILTER CUTOFF FRE



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)UENCY)

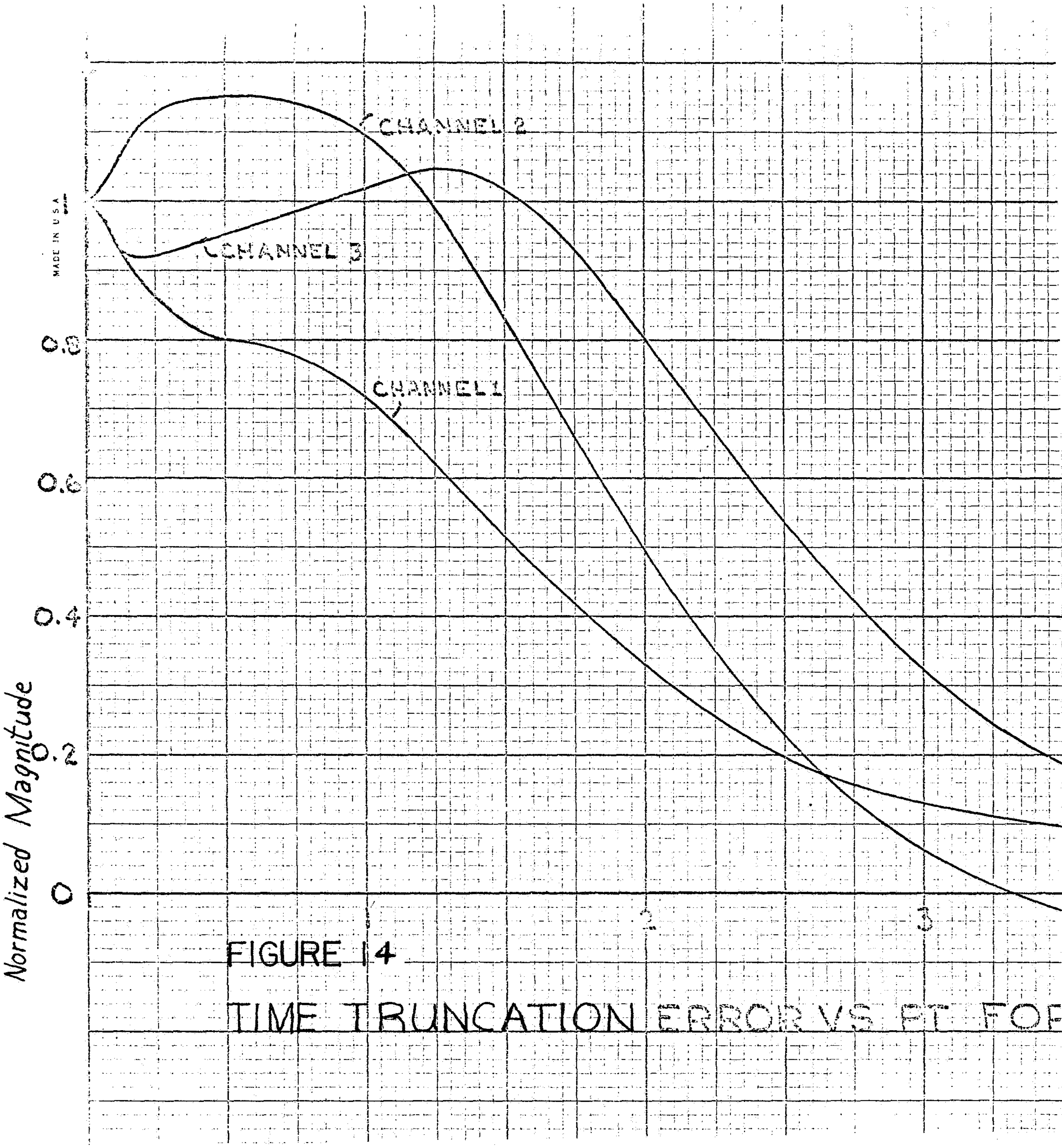
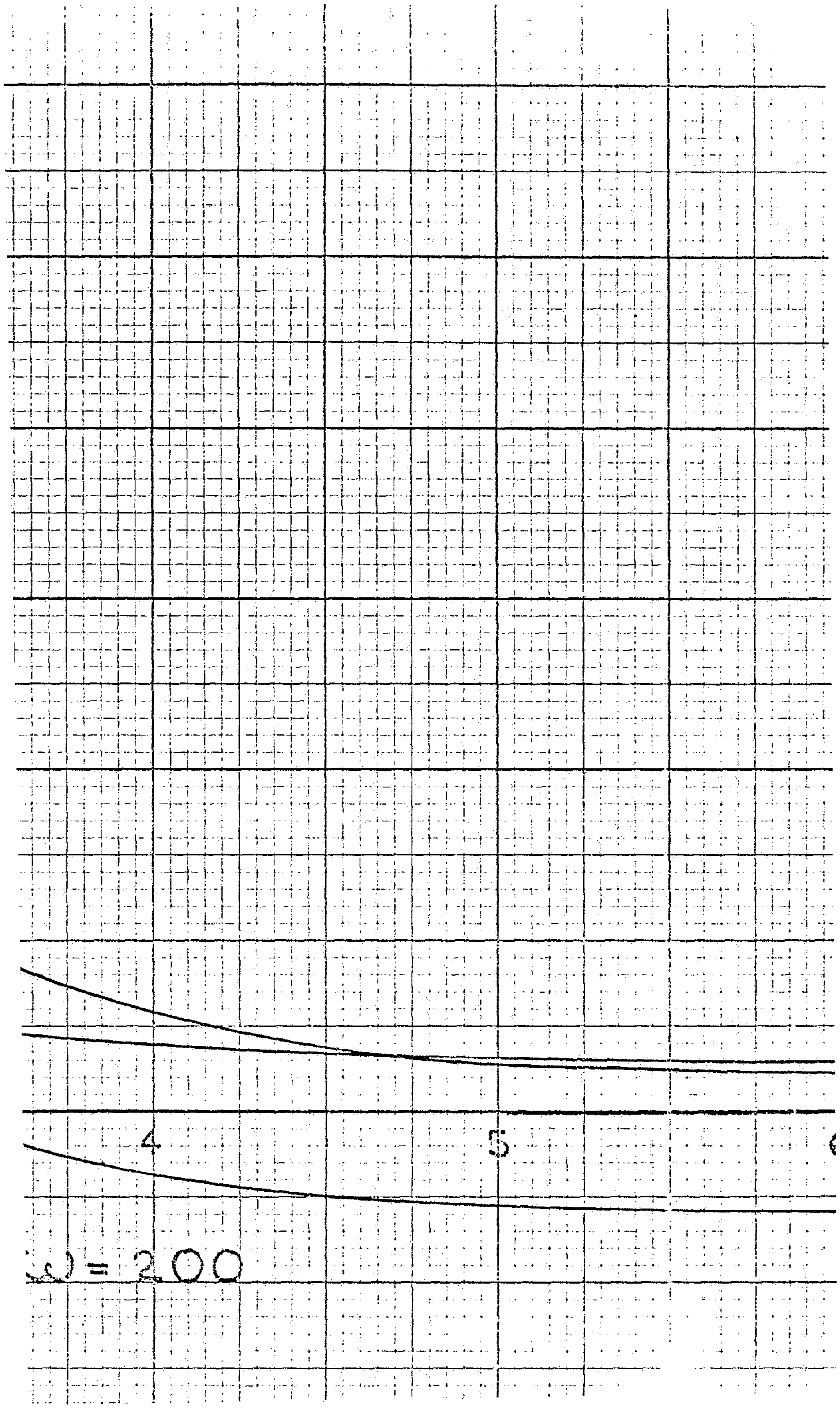


FIGURE 14

TIME TRUNCATION ERROR VS PT FOR


4
5
= 200

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